

# EXPLOSIVE VOLCANIC ERUPTIONS: FROM FRAGMENTATION TO ERUPTION COLUMNS. AN INTEGRATED MODELLISTIC-EXPERIMENTAL APPROACH

FABIO DIOGUARDI

Dipartimento di Scienze della Terra e Geoambientali, Università “Aldo Moro” di Bari, Via E. Orabona 4, 70125 Bari, Italy

## INTRODUCTION

In the last decades a big effort has been dedicated to the topic of explosive volcanic eruptions, as they represent one of the most spectacular and hazardous phenomena in nature and, when their magnitude is very large, they can influence the climate on a global scale. Up to now, many open questions in the knowledge of the mechanisms characterizing explosive eruptions still exist. A number of processes take place during an explosive eruption, which are characterized by the issue of a multiphase mixture (made of gas, magma droplets and/or pyroclastic particles) from the volcanic vent that drives the formation of an eruptive column. The production and motion of a multiphase mixture is therefore the distinguishing feature of an explosive eruption on all scales: if magma issues from the vent, an effusive eruption would occur. A multiphase mixture has to be produced in order to observe an explosive eruption, which means that magma needs to be transformed from a liquid to an accelerating mixture made of gas, particles and/or droplets. Moreover, for a magma to transform into a multiphase mixture, a fragmentation process has to take place. An explosive eruption can then be subdivided into three main processes:

- 1) fragmentation, *i.e.*, the change from a rising magma into an accelerating gas-particle mixture;
- 2) gas-particle conduit flow; *i.e.*, the motion of the multiphase mixture from the fragmentation level to the volcanic vent;
- 3) eruption column, that forms when the mixture issues from the conduit and it can be of very different sizes and shapes. Different eruptive regimes can be recognized, which actually constitute a continuum spectrum of sizes and scenarios, each one characterized by different eruption columns that determine different impacts on the territory.

This PhD thesis focuses on all the three processes from an experimental and numerical modeling point of view. After a detailed review on the past and present volcanological literature, new experimental models for magma fragmentation, gas-particle flow at conduit vent and eruptive scenarios are discussed. Finally, a numerical analysis of gas-particle conduit flow characteristics is presented and a recalibration of particle-wall friction laws, based on the comparison of numerical results and experimental measurements, is proposed.

## EXPERIMENTAL MODELING ON BRITTLE FRAGMENTATION OF MAGMATIC MELTS

Brittle fragmentation experiments were performed during the PhD research activities in the Physikalisch Vulkanologisches Labor (Universität Würzburg, Germany). The experimental setup for the so called “blowout” experiments was presented for the first time in Büttner *et al.* (2006) (although it was a modification of the setup used for investigating the magma-water interaction described in Büttner *et al.*, 2002), where its application for studying the fragmentation characteristics of trachytic melts from Phlegrean Fields was discussed. The new experiments were performed on shoshonitic melts from Stromboli 2002-2003 eruption and on phonolitic material coming from Mercato deposits (Vesuvius). The experimental configuration was the same used in 2006, in which a real magmatic melt, obtained by remelting volcanic rocks by magnetic induction, was stressed by an expanding pressurized gas, causing the deformation of the melt up to its critical strain rate, at which it fragments. The purposes are different: in Büttner *et al.* (2006) the critical strain rates and the energy dissipation in the fragmentation process were investigated. In this thesis, a new method for the calculation of the kinetic energy release at fragmentation is developed and presented. This physical parameter is crucial, as it marks the link

between the fragmentation and the onset of the gas-particle mixture motion in the volcanic conduit, which in turn influences the eruption column dynamics. The kinetic energy was obtained by combining the conservation of energy principle and a detailed analysis of the recorded pressure and force signals. The method consisted in identifying the time intervals at which each energy dissipation process is consumed, thus insulating the kinetic energy term (Fig. 1).

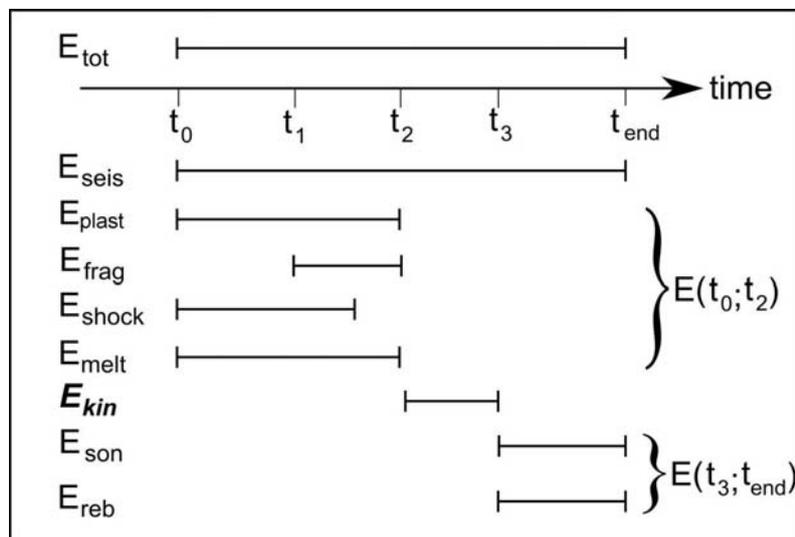


Fig. 1 - Chronology of energy dissipation during a brittle fragmentation experiment.  $E_{kin}$  is the wanted quantity and is highlighted. It is the only energy term dissipated in the time interval  $[t_2; t_3]$ ; thus, as the total energy  $E_{tot}$  is known and by determining  $E(t_0; t_2)$  and  $E(t_3; t_{end})$ , it can be readily obtained by subtraction.

The critical times that bound the energy dissipation intervals were obtained by searching for specific features in the recorded signals.

By normalizing the kinetic energy to the mass of the produced particles, it is possible to define the specific kinetic energy ( $SKE$ ), which is a measure of the average initial velocity of the particles  $v_{part}$  that proved to be a material dependent property.

The three tested samples were characterized by very different mechanical properties. The samples coming from Stromboli were of two types: dense particles (scoria with an initial vesicularity of about 30%) and highly vesiculated fragments (~ 80%), known as “golden pumices” (Rosi *et*

*al.*, 2006). Mercato sample showed an intermediate vesicularity (~ 50%; Mele *et al.*, 2011). Consequently, meaningful differences were observed during the experimental runs and in the model results. Stromboli dense material was always very difficult to break, which is in agreement with what observed in actual eruptions: a poorly vesiculated basaltic melt usually feeds an effusive eruption, as it fragments very rarely. Anyway, it was possible to fragment this melt, at least partially, and then to measure the wanted parameters. On the contrary, Stromboli “golden pumices” and Mercato melts were fragmented more easily and efficiently. As a consequence, Stromboli dense melt produced a lesser amount of fragments than those produced by Stromboli golden pumice and Mercato materials. The  $SKE$  released to particles produced by the more resistant melts was always higher than that of the less resistant ones: Stromboli dense material was characterized by a  $SKE$  value of about  $10.000 \text{ J kg}^{-1}$ , much larger than the  $SKE$  of Mercato ( $3.800 \text{ J kg}^{-1}$ ) and Stromboli “golden pumice” ( $3.100 \text{ J kg}^{-1}$ ). This result was confirmed by the initial particle velocities measured via high-speed video analysis: Stromboli dense particles moved at velocities between  $120$  and  $150 \text{ m s}^{-1}$ , much faster than Mercato (~  $90 \text{ m s}^{-1}$ ) and Stromboli “golden pumice” (~  $80 \text{ m s}^{-1}$ ). This effect can be explained by the larger amount of elastic energy stored by the dense melt before it reached the fragmentation threshold. From the results it can be concluded that, if a basaltic poorly vesiculated melt fragments, it tends to produce few particles that move at very high velocities; on the contrary, an intermediate and/or a highly vesiculated melt generates a large amount of particles that move slower than those produced by dense melts. More experimental runs are needed for improving the presented energetic method, enlarging the dataset of tested magmatic compositions and studying with more detail the influence of specific material parameters (vesicularity, crystal content, rheology, etc.) on the fragmentation process.

AN EXPERIMENTAL STUDY ON CONDUIT FLOW AND ERUPTION COLUMN DYNAMICS

An experimental model for the prediction of the eruptive mixture exit velocity from the volcanic vent is presented. The model was designed for linking the exit conditions to the magma fragmentation characteristics (particle size, kinetic energy release, etc.), and it was developed from the data recorded during large scale experiments on the dynamics of explosive eruptions (Dellino *et al.*, 2007, 2010a, 2010b). The experiments were designed for measuring physical quantities that are impossible to investigate in actual eruptions and to use real volcanic particles, whose coupling with the driving gas is strongly influenced by their morphology (Dellino *et al.*, 2005). In the experiments, pressurized gases were coupled to real volcanic particles in a vertical conduit in order to develop a multiphase mixture moving upwards, issuing from the conduit, and creating an eruptive column.

The experiments proved to be of large scale, as the Reynolds number of the formed eruptive mixtures was calculated to be in the same range expected for actual explosive eruptions. They were able to reproduce all the possible scenarios on different scales: convective plumes, collapsing columns, radially expanding jets and all the transitional behaviours. The occurrence of each of these scenarios was found to be strictly dependent on the specific mechanical energy (*SME*), which is the ratio between the total mechanical energy of the pressurized gas  $E_{tot}$  and the mass of particles  $m$ . When  $SME < 1.5 \text{ kJ kg}^{-1}$ , a collapsing column was always generated; if  $SME > 2.6 \text{ kJ kg}^{-1}$  a convective plume developed. In all the other cases, intermediate transitional columns were created.

The multiphase mixture exit velocity  $w_{ex}$  was monitored in each experimental run *via* high-speed video cameras positioned around the conduit. A correlation between the kinetic energy per unit mass and other measurable parameters was searched and a linear relationship was found (eq. 1; Fig. 2):

$$\frac{w_{ex}^2}{2} = 16.564 + 0.3115 \left( \frac{D}{L} \frac{E_{fragexp}}{P_{atm} V_P d_{PNrm} (P_{exit}/P_{atm})^{2.5}} \frac{E_{exit}}{m} \right) \quad (1)$$

where  $D$  and  $L$  are the conduit diameter and length, respectively,  $E_{fragexp}$  is the energy coupled to the particles in the conduit before they start to move (a quantity analogous to the kinetic energy release in the fragmentation experiments),  $E_{exit}$  is the energy dissipation in the movement of the mixture up to conduit vent,  $P_{atm}$  is the atmospheric pressure,  $V_P$  is the volume of particles,  $d_{PNrm}$  is the average particle diameter normalized by 1 mm,  $P_{exit}$  is the static pressure of the mixture at conduit vent, and  $m$  is the mass of particles.

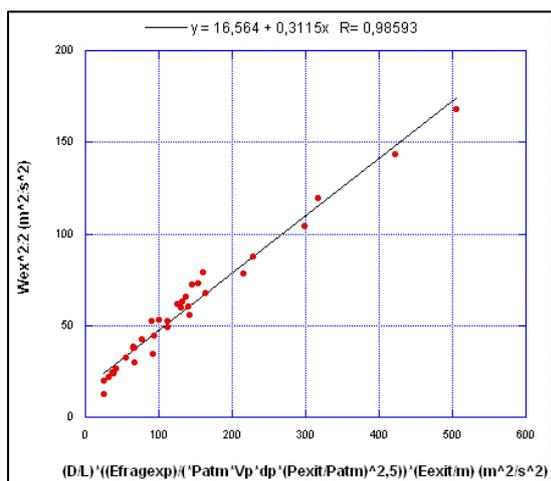


Fig. 2 - Diagram showing data correlation for the exit velocity model.

The law was designed to be linear and with the proper physical dimensions, in order to be readily applicable to actual cases.

The *SME* was proved to be a good regime parameter, but we searched for other parameters that are closely linked to the predicted exit velocity and other physical quantities which are easily to hypothesize in actual eruptions. In particular, two parameters were searched, one discriminating convective plumes and collapsing columns, the other one vertically evolving columns and radially expanding jets. The first one was obtained starting by theoretical considerations about the occurrence of specific eruptive scenarios, placing on the numerator the physical quantities which favor the development of a convective plume and on the denominator those favoring the formation of a collapsing column. Due to its physical dimensions ( $s^{-1}$ ), the so obtained regime parameter was

named “vorticity factor”  $\Omega$ :

$$\Omega = \frac{2w_{ex}}{R\varepsilon_p} \quad (2)$$

where  $R$  is the conduit radius and  $\varepsilon_p$  is the particle concentration. The critical value discriminating between convective plumes and collapsing columns was determined by calculating the vorticity factor for each experimental run and was found to be equal to  $500 \text{ s}^{-1}$  (thus  $\Omega > 500 \text{ s}^{-1}$  for the convective plumes,  $\Omega < 500 \text{ s}^{-1}$  for the collapsing columns). Furthermore, the ratio between the static exit overpressure of the gas-particle mixture ( $P_{over}$ ) and its dynamic pressure ( $P_{dyn}$ ) proved to be a regime parameter, called “overpressure factor”  $\Gamma$ , distinguishing between radially expanding jets and vertically directed columns:

$$\Gamma = \frac{P_{over}}{P_{dyn}} \quad (3)$$

A critical value of 0.3 was determined from the experiments ( $\Gamma > 0.3$  for radially expanding jets,  $\Gamma < 0.3$  for vertical columns).

The application of the empirical model for the prediction of exit velocity  $w_{ex}$  (eq. 1), together with the regime parameters  $\Omega$  and  $\Gamma$ , allowed to create regime diagrams for the prediction of the most likely scenarios given certain hypothesized conditions in an actual eruption. An example is displayed in Fig. 3.

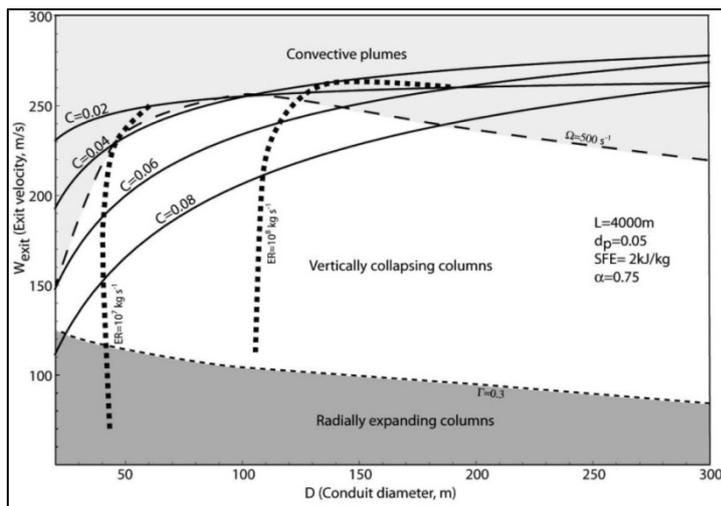


Fig. 3 - Regime diagram showing the stability fields of the three main eruptive regimes, by fixing the parameters, shown on the right, and varying the average particles concentration  $C$  (black lines). The thick dashed lines represent eruption rates, ER, of  $10^7$  and  $10^8 \text{ kg s}^{-1}$ , respectively.

In this diagram the exit velocity is plotted vs. the conduit diameter, by fixing the parameters listed in the inset and varying one parameter (the average particle concentration in this case). The curves of the critical values of the regime parameters could then be calculated and drawn, thus defining the stability fields of the three main eruptive regimes, with the hypothesized conditions.

The presented experimental model was proved to be consistent with the volcanological literature and helpful for the prediction of the main eruptive regimes. If the magma properties at fragmentation (particle size, specific fragmentation energy, gas volume, and pressure) and conduit geometry are known, they can be used for predicting the eruptive regime. They can be also

useful tools for modelers who want to perform numerical simulations of eruptive columns starting from the conduit exit. For all these reasons, the model results a useful tool for hazard assessment, as it provides information on the possible eruptive style of an explosive eruption and, therefore, the type of impact of the eruption on the territory.

## PARAMETRIC NUMERICAL INVESTIGATIONS ON VERTICAL GAS-PARTICLE CONDUIT FLOW

A numerical model was implemented for simulating the vertical gas-particle conduit flow in the large scale experiments (Dellino *et al.*, 2007, 2010a, 2010b). The aim of this model was to study the influence of crucial multiphase fluid dynamics parameters (*e.g.*, interphase drag coefficient, wall-particles frictions, and particles shape factor) on the conduit flow. A lot of empirical models for the evaluation of these parameters are available, which were mainly developed for industrial engineering purposes. These models are usually obtained by experiments whose physical ranges are very different from those of volcanological interest (Rautiainen & Sarkomaa, 1998). For this reason, the applicability of these empirical laws to the simulation of explosive eruptions conduit dynamics needs a careful testing. Here these models were tested and re-calibrated for obtaining suitable parameters for volcanic eruptions, by the combined use of the large scale experiments and the numerical simulations.

The motion of gas (nitrogen) and particles (volcanic ash) in the experimental conduit was modeled with a one-dimensional, steady, eulerian-eulerian non-homogeneous approach. The eulerian-eulerian approach treats the phases involved in the multiphase flow as interpenetrating continua and is suitable for highly concentrated flows (average volumetric particle concentrations exceeding 0.1). The flow is assumed to be isothermal, due to the typical time scales of the conduit flow in the experiments (less than 1 s) and the very low thermal conductivity of the volcanic particles ( $< 1 \text{ W m}^{-1} \text{ K}^{-1}$ , Büttner *et al.*, 2000). Thus, only the equation of conservations of mass and momentum had to be solved, together with the state equation and the conservation of total volume equation, which states that the sum of particle and gas volume fractions adds up to one. The equations constitute a system of ordinary differential equations that was solved via an adaptive step-size Runge-Kutta method.

The particle-wall friction term, together with the interphase drag, particle shape factor, initial void fraction, etc., were varied parametrically. In this way more than 500 parameters combinations, which actually represent different models, were tested. For each model, all the experimental runs were simulated for obtaining the velocity and volumetric concentration trends for the two phases. In order to evaluate the goodness of the solution, the computed exit velocity of the particles  $w_{exsim}$  was compared with the exit velocities measured in the experiments. For each experimental run the error was evaluated by dividing the deviation  $w_{exsim} - w_{ex}$  by  $w_{ex}$ :  $(w_{exsim} - w_{ex}) / w_{ex}$ . The average of these deviations was then calculated for having information of the overall

goodness of each tested model. Many models proved to be successful for the simulation of all the experimental runs; *e.g.*, in Fig. 4 the calculated exit velocity for each experiment is plotted vs. the measured exit velocity.

A number of considerations about the tested parameters were possible. In particular, the particle shape factor had a strong influence on the simulated velocity of the two phases, due to its influence on the mechanical coupling between the phases: the larger the shape factor (thus the more similar the shape of the particle to that of the sphere), the larger the velocity difference between gas and particles. Furthermore, three different laws for the calculation of the interphase drag coefficient were tested: Wen & Yu (1966), Syamlal & O'Brien (1989), Gidaspow *et al.* (1992). Results obtained with any of these laws were

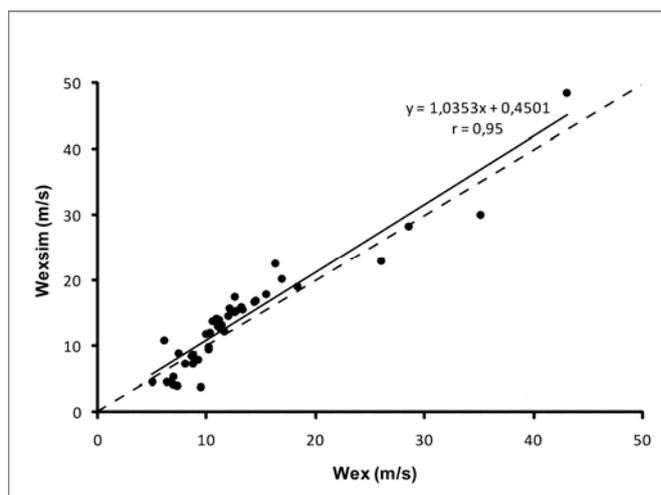


Fig. 4 - Simulated vs. measured exit velocity for all the experiments. The dashed line represents the perfect agreement, the solid line is the linear regression with the equation and the correlation coefficient  $r$  in the inset.

always similar, with only that of Gidaspow *et al.* (1992) being more suitable for the more concentrated flows.

A particular care was dedicated to the study of the influence of the different empirical laws for the particle-wall friction factor, as this choice proved to have the largest influence on the model results among the studied parameters. The tested laws are reported in Table 1.

Table 1 - List of the considered particle-wall friction factor laws.

Empirical law	Reference
$f_p = 0.0032 \frac{\varepsilon_p}{\varepsilon_G} \left( \frac{\varepsilon_p w_p}{u-v} \right)^{-0.979}$	Yang (1978)
$f_p = 0.0029 \frac{(gD)^{0.5}}{\nu}$	Konno & Saito (1969)
$f_p = 0.0017 \frac{\varepsilon_p w_p}{\varepsilon_G^3 u_{sup}} \left( \frac{\varepsilon_p}{(u-v)/w_p} \right)^{-1.5}$	Garić <i>et al.</i> (1995)
$f_p = 0.01 \frac{(u_{sup}/\nu)^2}{w_p/(gD)^{0.5}}$	Matsumoto <i>et al.</i> (1986)
$f_p = \frac{55.5D^{1.1}}{\nu^{0.65} d_p^{0.26} \rho_p^{0.91}}$	Klinzing & Mathur (1981)
$f_p = 12.2 \frac{\varepsilon_p}{\varepsilon_G^3 \nu}$	Breault & Mathur (1989)

The deviations due to the use of a particular particle-wall friction law were calculated. It was found that the more concentrated flow (those feeding collapsing column experiments) required different laws from the more dilute cases (convective plume experiments). In particular, for the collapsing column runs the average deviations were similar for all the tested laws, whereas for the convective plume runs Klinzing & Mathur (1981) and Yang (1978) laws proved to be the best ones. By combining the experimental measurements and the numerical model, some of the considered friction laws were recalibrated in order to reduce the deviations and to better fit the experimentally measured exit velocities. For collapses, the best results were obtained with the modified Yang (1978) law:

$$f_p = 0.0082 \frac{\varepsilon_p}{\varepsilon_G} \left( \frac{\varepsilon_p w_p}{u-v} \right)^{-0.979} \quad (4)$$

with an average deviation over all the experiments of about 10% and a correlation coefficient between simulated and measured exit velocities close to 0.9. For plumes, the modified version of Konno & Saito (1969) law:

$$f_p = 0.0049 \frac{(gD)^{0.5}}{\nu} \quad (5)$$

resulted in very good performances, with an average deviation of about -6% and a correlation coefficient of 0.95.

Despite some limitations, such as steady state flow and monodimensional particles, the model proved to be a powerful tool for a detailed parametric study based on experimental measurements. The particle shape factor confirmed to be a crucial parameter in multiphase flow dynamics, as it influences the interphase momentum coupling; on the other hand the different laws for the calculation of the interphase drag provided very similar results. Finally, the choice of the different particle-wall friction laws proved to have a great influence on

the solutions. As a consequence, each flow regime (highly concentrated and dilute) needed the application of a different friction law.

Thanks to the availability of experimental measurements, some of these laws were re-calibrated in order to obtain new laws that are more suitable for the simulation of vertical 1D gas-particle flows. The next step will be the implementation of an unsteady model for better constraining the initial stages of the motion and the possible unsteady features of the gas-particle conduit flow in an explosive eruption.

## CONCLUSION

The PhD thesis focused on the main processes constituting an explosive volcanic eruption: magma fragmentation, gas-particle conduit flow and eruption column dynamics. The three processes were investigated with an experimental and numerical point of view. Experiments on brittle magmatic fragmentation were conducted by employing real magmatic melts for the evaluation of the kinetic energy released to the generated fragments. This quantity is crucial as it determines the initiation of motion of the eruptive mixture in the volcanic conduit. It was found that the specific kinetic energy is strictly dependent on the melt properties (magma composition and vesicularity). Large-scale experiments on conduit flow and eruption column dynamics were also performed and an experimental model linking gas-particle flow characteristics at conduit vent to the physical properties of magma at fragmentation was obtained by experimental measurements. Regime parameters allowing to distinguish between the different eruptive column regimes (collapsing columns, convective plumes, etc.) were also derived and used, together with the conduit exit model, for constraining the possible eruptive regimes to some crucial physical quantities that are easy to measure or hypothesize in real volcanic eruptions. Finally, a numerical model for the simulation of the gas-particle conduit flow was applied for determining the influence of certain crucial multiphase fluid dynamics parameters (*e.g.*, particle shape factor, interphase drag, particle-wall friction). In particular, the possibility to use the available particle-wall friction factor laws in a volcanological context was verified by applying the numerical model to the large-scale experiments and it was found that only few of these laws are suitable for quantifying the friction of real volcanic particle with the conduit. These laws were eventually recalibrated in order to better fit the experimental measurements and, as the experiments proved to be representative of actual eruptions, to be applicable in the simulations of real gas-particle conduit flows.

## REFERENCES

- Breault, R.W. & Mathur, V.K. (1989): High-velocity fluidized-bed hydrodynamic modeling. 2. Circulating bed pressure drop modeling. *Ind. Engin. Chem. Res.*, **28**(6), 688-693.
- Büttner, R., Zimanowski, B., Lenk, C., Koopmann, A., Lorenz, V. (2000): Determination of thermal conductivity of natural silicate melts. *Appl. Phys. Lett.*, doi: 10.1063/1.1311815.
- Büttner, R., Dellino, P., La Volpe, L., Lorenz, V., Zimanowski, B. (2002): Thermohydraulic explosions in phreatomagmatic eruptions as evidenced by the comparison between pyroclasts and products from Molten Fuel Coolant Interaction experiments. *J. Geophys. Res.*, **107**, 2277, doi: 10.1029/2001JB000511.
- Büttner, R., Dellino, P., Raue, H., Sonder, I., Zimanowski, B. (2006): Stress-induced brittle fragmentation of magmatic melts: Theory and experiments. *J. Geophys. Res.*, **111**, B08204, doi: 10.1029/2005JB003958.
- Dellino, P., Mele, D., Bonasia, R., Braia, G., La Volpe, L., Sulpizio, R. (2005): The analysis of the influence of pumice shape on its terminal velocity. *Geophys. Res. Lett.*, **32**, L21306, doi: 10.1029/2005GL023954.
- Dellino, P., Zimanowski, B., Büttner, R., La Volpe, L., Mele, D., Sulpizio, R. (2007): Large-scale experiments on the mechanics of pyroclastic flows: Design, engineering, and first results. *J. Geophys. Res.*, **112**, B04202, doi: 10.1029/2006JB004313.
- Dellino, P., Dioguardi, F., Zimanowski, B., Büttner, R., Mele, D., La Volpe, L., Sulpizio, R., Doronzo, D.M., Sonder, I., Bonasia, R., Calvari, S., Marotta, E. (2010a): Conduit flow experiments help constraining the regime of explosive eruptions. *J. Geophys. Res.*, **115**, B04204, doi: 10.1029/2009JB006781.

- Dellino, P., Büttner, R., Dioguardi, F., Doronzo, D.M., La Volpe, L., Mele, D., Sonder, I., Sulpizio, R., Zimanowski, B. (2010b): Experimental evidence links volcanic particles characteristics to pyroclastic flow hazard. *Earth Planet. Sci. Lett.*, **295**, 314-320, doi: 10.1016/j.epsl.2010.04.022.
- Garić, R.V., Grbavčić, Ž.B., Jovanović, S.Dj. (1995): Hydrodynamic modeling of vertical non-accelerating gas-solids flow. *Powder Technol.*, **84**, 65-74.
- Gidaspow, D., Bezburuah, R., Ding, J. (1992): Hydrodynamics of Circulating Fluidized Beds, Kinetic Theory Approach. In: "Fluidization VII", Proceed. 7<sup>th</sup> Engineering Foundation Conference on Fluidization, 3-8 May 1992, Brisbane, Australia, 75-82.
- Klinzing, G.E. & Mathur, M.P. (1981): The dense and extrusion flow regime in gas-solid transport. *Can. J. Chem. Engin.*, **59**, 590-594.
- Konno, H. & Saito, S. (1969): Pneumatic conveying of solids through straight pipes. *J. Chem. Engin. Japan*, **2**, 211-217.
- Matsumoto, S., Harakawa, H., Suzuki, M., Ohtani, S. (1986): Solid particle velocity in vertical gaseous suspension flows. *Int. J. Multiphase Flow*, **12**, 445-458.
- Mele, D., Sulpizio, R., Dellino, P., La Volpe, L. (2011): Stratigraphy and eruptive dynamics of a pulsating Plinian eruption of Somma-Vesuvius: the Pomice di Mercato (8900 years B.P.). *Bull. Volcanol.*, doi: 10.1007/s00445-010-0407-2.
- Rautiainen, A. & Sarkomaa, P. (1998): Solid friction factors in upward, lean gas-solid flows. *Powder Technol.*, **95**, 25-35.
- Rosi, M., Bertagnini, A., Harris, A.J.L., Pioli, L., Pistolesi, M., Ripepe, M. (2006): A case history of paroxysmal explosion at Stromboli: Timing and dynamics of the April 5, 2003 event. *Earth Planet. Sci. Lett.*, **243**, 594-606.
- Syamlal, M. & O'Brien, T.J. (1989): Computer simulation of bubbles in a fluidized bed. *AIChE Symp. Series*, **85**, 22-31.
- Wen, C.Y. & Yu, Y.H. (1966): Mechanics of fluidization. *Chem. Engin. Prog. Symp. Series*, **62**, 100-111.
- Yang, W.C. (1978): A correlation for solid friction factor in vertical pneumatic conveying lines. *AIChE J.*, **24**, 548-552.